

technical note

## How well are clinical gross tumor volume DVHs approximated by an analytical function?

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The dose heterogeneity in the tumor is often described as being normally distributed. Besides the normal distribution we propose the Fermi function describing Fermi statistics as a possible dose heterogeneity descriptor. In order to demonstrate the adequacy of the proposed functions as dose distribution descriptors 30 clinical gross tumor volume (GTV) dose-volume histograms (DVHs) are gathered and fit with the examined functions.

Key words: dose-volume histograms; gross tumor volume; Gaussian and Fermi statistics

In order to theoretically investigate a given tumor control probability (TCP) model for the case of heterogeneous irradiation, it is often necessary to simulate tumor dose-volume histograms (DVHs) that closely resemble clinical ones. Some authors<sup>1-3</sup> have assumed that tumor dose inhomogeneities are normally distributed around the target dose. In this case the integral DVH, *iDVH*, is represented by the erfc function:

$$iDVH: v(d|\mu, \theta) = 0.5 \operatorname{erfc} \left( \frac{1}{\theta \sqrt{2\pi}} (d - \mu) \right) \quad [1]$$

where  $v$  is the relative tumor volume irradiated to a maximum dose  $d$ ,  $\mu$  is a parameter

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corresponding to the mean (target) dose delivered to the tumor, and  $\theta$  is a parameter related to the slope of the erfc function. However, no investigations of how well this function describes the clinical tumor DVHs are reported in the literature.

We propose the parallel use of the Fermi statistics function for the description of clinical tumor DVHs:

$$iDVH: v(d|\mu, \theta) = \frac{1}{1 + \exp \left( \frac{d - \mu}{\theta} \right)} \quad [2]$$

This function describes the filling up of free energy levels in a Fermi system. The parameters  $d$ ,  $\mu$  and  $\theta$  have the same meaning as in eq. [1].

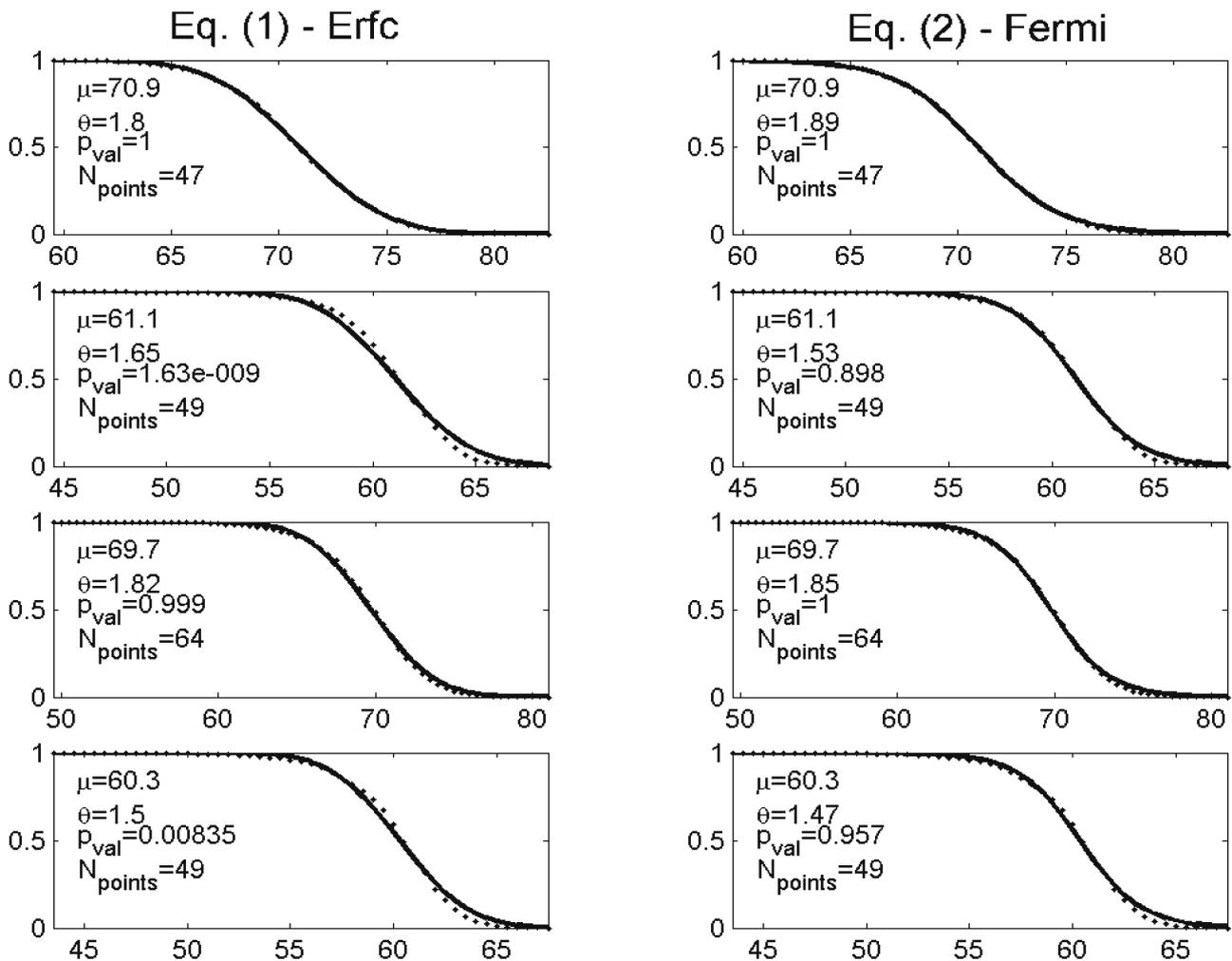
To investigate this problem, we gathered 30 clinical gross tumor volume (GTV) DVHs for different treatment sites – lung, head & neck, prostate, etc., that were either

obtained in the treatment planning process at the Cross Cancer Institute (CCI) or reported in the literature.<sup>4-14</sup> They were fit with the erfc [Eq. 1] and Fermi [Eq. 2] functions.

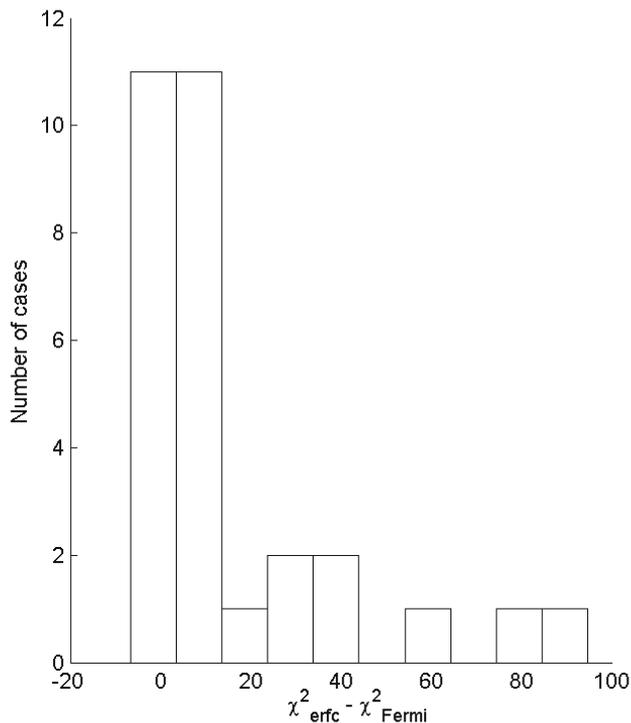
The fit was performed using the  $\chi^2$  criterion for goodness of fit, presuming a log-log-normal distribution for the integral DVHs. Correspondingly, the function to be minimized is:

$$\chi^2 = \sum_i \left[ \frac{-\ln(-\ln(v_{theoretical}^i)) + \ln(-\ln(v_{experimental}^i))}{\sigma_{\log\ experimental}^i} \right]^2 ; \sigma_{\log\ experimental}^i = \frac{-\sigma_{\experimental}^i}{v_{\experimental}^i \ln(v_{\experimental}^i)}$$

The log-log form of the  $\chi^2$  criterion was used to account for the fact that an integral DVH is defined in the interval [0,1], while the standard  $\chi^2$  criterion deals with normally distributed random variables defined in  $(-\infty, +\infty)$ .<sup>15</sup> The experimental error is  $\sigma_{\experimental}$ , which unfortunately is not reported in the literature. Therefore, we substituted  $\sigma_{\experimental}$  with a percentage band



**Figure 1.** Fits to four clinical (head & neck) DVHs from CCI with the erfc function – a) and with the Fermi function – b) for the case of a 2% error band ( $\sigma_{\experimental} = 2\%$ ). On each subplot the p-value of the fit are shown, along with the statistics (number of data points,  $N_{points}$ ) and the corresponding best fit values of the model parameters ( $\mu$  and  $\theta$ ).



**Figure 2.** Distribution of the difference of  $\chi^2$  values for the fits obtained with the erfc and the Fermi functions,  $(\chi^2_{erfc} - \chi^2_{Fermi})$ , built on the bases of fits to 30 clinical DVHs.

in which a statistically acceptable fit could be obtained. We initially presumed a 2% error band and found that in 24 out of 30 cases Eq. [2] produced acceptable fits, while Eq. [1] produced acceptable fits in 19 out of 30 cases. We also investigated a 4% error band and found out that Eq. [2] produced statistically acceptable fits in all considered cases, while Eq. [1] produced poor fits in 2 of all cases. As an illustration, fits of four head & neck DVHs obtained at CCI with the erfc function [Eq. 1] and with the Fermi function [Eq. (2)] are shown in Figure 1a and Figure 1b, respectively. The fits correspond to the case of  $\sigma_{\text{experimental}} = 2\%$ . As can be seen from the obtained  $\chi^2$  and p-values (shown in each subplot of Figure 1), the Fermi function produces excellent fits in all four cases, while the erfc function produces a poor fit in one case. It can be therefore concluded that the Fermi function describes clinical integral DVHs better than the erfc function. To fur-

ther illustrate this conclusion a distribution of the difference of the  $\chi^2$  values of the fits obtained with the two proposed functions is shown in Figure 2. This distribution is constructed on the bases of the 30 clinical DVHs used in this study. As can be seen from Figure 2, the average of the distribution of  $(\chi^2_{erfc} - \chi^2_{Fermi})$  is greater than zero, showing that in most cases the Fermi function indeed produces better fits to clinical DVHs than the erfc function.

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